MANIFESTLY SUPERPOINCARÉ-COVARIANT QUANTIZATION OF THE GREEN-SCHWARZ SUPERSTRING

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The Green–Schwarz (GS) superstring is reformulated in a physically equivalent way by embedding it into a larger system containing additional fermionic string- as well as bosonic harmonic variables and possessing additional gauge invariances. The main feature of the new GS superstring system is that it contains covariant and functionally independent first-class constraints only. This allows straightforward application of the BFV–BRST formalism for a manifestly superPoincaré-covariant canonical quantization. The corresponding BRST charge turns out to be of second rank and, therefore, the BFV–BRST action contains fourth-order ghost terms.

1. Motivation

This letter is aimed at providing a solution to the problem of manifestly superPoincaré-covariant canonical quantization of superstrings in the Green–Schwarz (GS) formulation [1]. The present study is a direct continuation and elaboration of the formalism developed in ref. [2]. There a new type of $D=10$ harmonic superspace (cf. also ref. [3]) with two generations of pure-gauge bosonic harmonic coordinates was introduced. It allowed to reduce the GS superstring to a system whose constraints were both Lorentz-covariant and functionally independent (irreducible, according to the terminology of Batalin–Fradkin–Vilkovisky (BFV) [4,5]). Thus, the long-standing problem [6] of covariant and irreducible separation of the first- and second-class fermionic constraints in the original GS superstring action was solved [#1]. However, in order to achieve simple canonical Dirac brackets among the superstring coordinates (due to the presence of the covariant second-class constraints) we were obliged in ref. [2] to impose covariant gauge-fixing of the fermionic κ-gauge invariance (the first-class part of the fermionic GS constraints). Thus part of the superPoincaré algebra (some translations and supersymmetry transformations), although acting in a manifestly Lorentz-covariant way, became non-linearly realized.

In this letter we propose to further enlarge the GS superstring. The new system has additional gauge invariance and contains besides the pure gauge bosonic harmonic coordinates of refs. [2,3] additional pure gauge fermionic string variables. A crucial property of this enlarged GS superstring is that it possesses covariant and irreducible first-class constraints only. Now, the application of the covariant BFV–BRST quantization procedure [4,5] becomes straightforward and the whole superPoincaré invariance is manifestly preserved. The only unusual feature is that the corresponding BRST charge contains higher-order (fifth power) ghost terms, i.e., it is of rank two according to the BFV terminology [4,5].

In fact, there exists already in the literature a covariant superstring system [10] which contains first-class constraints only. However, these constraints form a reducible set with an infinite level of reducibility [9,11,12]. The formalism proposed in ref. [11] for eliminating the higher ghost generations (ghosts for ghosts) in the BFV

[#1] The covariant separation proposed in refs. [7,8] leads in fact to reducible (functionally dependent) sets of constraints. Application of the general BFV procedure [4] to treat these reducible constraints would force the level of reducibility to be ∞ [9] which renders the formalism of refs. [7,8] intractable.

treatment of the Siegel superstring [10] explicitly breaks Lorentz invariance at the level of the BFV–BRST action since it introduces there constant light-like Lorentz vectors which are not dynamical degrees of freedom.

In the covariant harmonic superspace quantization [3] of $D=10$ $N=1, 2$ Brink–Schwarz (BS) superparticles [13] ("zero-mode" approximations of the GS superstring) we were able to convert the $8N$ covariant second-class fermionic constraints into an equivalent set of covariant $4N$ first-class fermionic constraints thus reformulating the $N=1, 2$ BS superparticle as a system with covariant first-class constraints only. Unfortunately, direct generalization of this procedure to the case of the GS superstring turned out to be impossible due to the complicated structure of the Poisson bracket (PB) relations among the fermionic superstring constraints [eqs. (12) and (13) below]. [In the point-particle limit the RHS of (13) is zero, and the RHS of (12) does not contain any fermionic constraints.]

Our present approach is based on an idea first proposed in ref. [14] in a different context and further elaborated in great detail and generality in ref. [15]. This idea is that any irreducible set of $2K$ bosonic (fermionic) second-class constraints may be converted into the same number of irreducible first-class constraints at the price of introducing additional pure gauge K bosonic (fermionic) degrees of freedom. The general formalism of ref. (15) cannot be directly applied to the GS superstring constraints [eqs. (12)–(17) below] since now we have both second- and first-class constraints and, moreover, the RHS of eq. (12) contains an unusual term quadratic in the constraints. Nevertheless, we find below an appropriate modification of all harmonic superstring constraints by adding terms containing additional fermionic string variables $\Xi^a_A(\xi)$ [see eqs. (20)–(23) below] in such a way that this modified system becomes first-class only.

2. Enlarged harmonic superstring action

The GS superstring action [1] in $D=10$ harmonic superspace $(X^\mu(\xi), \theta_{A\alpha}(\xi); u^a_\mu, V^{\pm 1/2}_\alpha)$, written in hamiltonian form, reads [2]

$$S=S_{\rm GS}+S_{\rm harmonic}\,, \tag{1}$$

$$S_{\rm GS}=\int \mathrm{d}\tau \int_{-\pi}^{\pi} \mathrm{d}\xi \left(\mathscr{P}_\mu \partial_\tau X^\mu + \sum_A p^\alpha_{\theta A} \partial_\tau \theta_{A\alpha} - \sum_A \Lambda_A T_A - \sum_A M_\alpha D^\alpha_A \right), \tag{2}$$

$$S_{\rm harmonic}=\int \mathrm{d}\tau \left(p^a_{u\mu} \partial_\tau u^\mu_a + p^{\mp 1/2\,\alpha}_v \partial_\tau v^{\pm 1/2}_\alpha - \Lambda_{ab} d^{ab} - \Lambda^{+-} d^{-+} - \Lambda^-_a d^{+a} \right). \tag{3}$$

Here $\theta_A=\theta_{A\alpha}$ $(A=1, 2)$ are two Majorana–Weyl (MW) spinors with equal (left-handed) chiralities (we are considering the case of type IIB GS superstring for definiteness). The bosonic harmonic variables entering the action (3) consist of the following objects:

(i) $v^{\pm 1/2}_\alpha$, two $D=10$ (left-handed) MW spinors,

(ii) u^a_μ, eight $(a=1, ..., 8)$ $D=10$ Lorentz vectors,

which satisfy the constraints

$$[v^{+1/2}_\alpha (\sigma^\mu)^{\alpha\beta} v^{+1/2}_\beta][v^{-1/2}_\gamma (\sigma_\mu)^{\gamma\delta} v^{-1/2}_\delta] = -1\,, \qquad u^a_\mu [v^{\pm 1/2}_\alpha (\sigma^\mu)^{\alpha\beta} v^{\pm 1/2}_\beta]=0\,, \qquad u^a_\mu u^{b\mu}=C^{ab}\,. \tag{4}$$

C^{ab} denotes the $D=8$ charge-conjugation matrix. The group $SO(8)\times SO(1, 1)$ acts on u^a_μ, $v^{\pm 1/2}_\alpha$ as an internal group of local rotations where u^a_μ transform as $SO(8)$ (s)-spinors whereas $v^{\pm 1/2}_\alpha$ carry charge $\pm 1/2$ under $SO(1, 1)$. Thus, the vectors u^a_μ together with the composite identically light-like vectors

$$u^\pm_\mu = v^{\pm 1/2} \sigma_\mu v^{\pm 1/2} \tag{5}$$

realize through (4) the coset space SO(1, 9)/SO(8)×SO(1, 1) [#2]. The light-like property of $u^{\pm}_{\mu}$ (5) is due to the well-known $D=10$ Fierz identity (see e.g. ref. [1])

$$(\sigma_{\mu})^{\alpha\beta}(\sigma^{\mu})^{\gamma\delta}+(\sigma_{\mu})^{\beta\gamma}(\sigma^{\mu})^{\alpha\delta}+(\sigma_{\mu})^{\gamma\alpha}(\sigma^{\mu})^{\beta\delta}=0\,. \tag{6}$$

The following notations for the constraints appearing in the actions (2) and (3) are used:

$$T_{A}\equiv\Pi^{2}_{A}+4\mathrm{i}(-1)^{A}D^{\alpha}_{A}\theta'_{A\alpha}\quad\text{(reparametrization constraints)}\,, \tag{7}$$

with

$$\Pi^{\mu}_{A}\equiv\mathscr{P}^{\mu}+(-1)^{A}[X'^{\mu}+2\mathrm{i}\theta_{A}\sigma^{\mu}\theta'_{A}]\,,$$

$$D^{\alpha}_{A}\equiv-\mathrm{i}p^{\alpha}_{\theta A}-[\mathscr{P}^{\mu}+(-1)^{A}(X'^{\mu}+\mathrm{i}\theta_{A}\sigma^{\mu}\theta'_{A})](\sigma_{\mu}\theta_{A})^{\alpha}\,, \tag{8}$$

$$D^{ab}=u^{a}_{\mu}\partial/\partial u_{\mu b}-u^{b}_{\mu}\partial/\partial u_{\mu a},\quad D^{-+}=\tfrac{1}{2}(v^{+1/2}_{\alpha}\partial/\partial v^{+1/2}_{\alpha}-v^{-1/2}_{\alpha}\partial/\partial v^{-1/2}_{\alpha})\,,$$

$$D^{+a}=u^{+}_{\mu}\,\partial/\partial u_{\mu a}+\tfrac{1}{2}(v^{-1/2}\sigma^{+}\sigma^{a}\partial/\partial v^{-1/2})\,, \tag{9}$$

where u^{+}_{μ} is as in (5), $\sigma^{\pm}\equiv\sigma^{\mu}u^{\pm}_{\mu}$, $\sigma^{a}\equiv\sigma^{\mu}u^{a}_{\mu}$ and D^{ab}, D^{-+}, D^{+a} are the first-quantized forms of the purely harmonic constraints d^{ab}, d^{-+}, d^{+a} in (3).

The action (1)–(3) and the constraints (7)–(9) are invariant under global spacetime supersymmetry transformations:

$$\delta_{\mathrm{ss}}X^{\mu}(\xi)=-\mathrm{i}\sum_{A}\varepsilon_{A}\sigma^{\mu}\theta_{A}(\xi),\quad \delta_{\mathrm{ss}}\theta_{A}(\xi)=\varepsilon_{A},\quad \delta_{\mathrm{ss}}v^{\pm1/2}_{\alpha}=\delta_{\mathrm{ss}}u^{a}_{\mu}=0\,.$$

In the hamiltonian framework the corresponding supersymmetry generators read

$$Q^{\alpha}_{A}=\int_{-\pi}^{\pi}\mathrm{d}\xi\,Q^{\alpha}_{A}(\xi)\,,\quad Q^{\alpha}_{A}(\xi)\equiv-\mathrm{i}p^{\alpha}_{\theta A}+[\mathscr{P}^{\mu}+(-1)^{A}(X'^{\mu}+\mathrm{i}\theta_{A}\sigma^{\mu}\theta'_{A})](\sigma_{\mu}\theta_{A})^{\alpha}\,.$$

All constraints in (2) and (3) are first class except D^{α}_{A} (8) which is a mixture of first and second class. Λ_{A}, ..., Λ^{-}_{a} in (2) and (3) are the corresponding Lagrange multipliers.

It was already stressed in ref. [2] that independence of the harmonic variables u^{a}_{μ}, $v^{\pm1/2}_{\alpha}$ on the string world-sheet parameter ξ does not spoil the reparametrization invariance of the harmonic GS superstring (1)–(3) since u^{a}_{μ}, $v^{\pm1/2}_{\alpha}$ are pure gauge degrees of freedom. Their crucial role is to covariantly and irreducibly disentangle the first- and the second-class parts of the fermionic constraints D^{α}_{A} (8):

$$D^{\alpha}_{A}=(\Pi^{+}_{A})^{-1}(\sigma^{b}v^{+1/2})^{\alpha}D^{+1/2}_{Ab}+(\Pi^{+}_{A})^{-1}(\not{\Pi}_{A}\sigma^{+}\sigma^{b}v^{-1/2})^{\alpha}G^{+1/2}_{Ab}\,, \tag{10}$$

where $\Pi^{+}_{A}\equiv v^{+1/2}\not{\Pi}_{A}v^{+1/2}$ is a Lorentz-scalar, or inversely

$$D^{+1/2\,a}_{A}=v^{+1/2}\sigma^{a}\not{\Pi}_{A}D_{A}\quad\text{(first-class generator of the }\kappa\text{-symmetry)}\,,$$

$$G^{+1/2\,a}_{A}=\tfrac{1}{2}(v^{-1/2}\sigma^{a}\sigma^{+}D_{A})\quad\text{(second class)}\,. \tag{11}$$

The PB relations among T_{A}, D^{α}_{A} in terms of (10) and (11) read

$$\begin{aligned}\{D^{+1/2\,a}_{A}(\xi),D^{+1/2\,b}_{B}(\eta)\}_{\mathrm{PB}}&=-2\mathrm{i}\delta_{AB}C^{ab}\Pi^{+}_{A}\,T_{A}\delta(\xi-\eta)\\&\quad-8(-1)^{A}\delta_{AB}\delta(\xi-\eta)[\psi^{+1/2\,a}_{A}D^{+1/2\,b}_{A}+\psi^{+1/2\,b}_{A}D^{+1/2\,a}_{A}-C^{ab}\psi^{+1/2\,c}_{A}D^{+1/2}_{Ac}]\\&\quad+8r^{acd}G^{+1/2}_{Ad}(\xi)[(-1)^{A}\delta_{AB}\delta'(\xi-\eta)]r^{b}{}_{c}{}^{e}G^{+1/2}_{Ae}(\eta)\,,\end{aligned} \tag{12}$$

$$\{D^{+1/2\,a}_{A}(\xi),G^{+1/2\,b}_{B}(\eta)\}_{\mathrm{PB}}=4(-1)^{A}\delta_{AB}\delta(\xi-\eta)R^{+1/2\,abd}_{A}G^{+1/2}_{Ad}\,, \tag{13}$$

[#2] A $D=10$ harmonic superspace of this type with elementary light-like vectors $u^{\pm}_{\mu}$ was earlier introduced in ref. [16].

$$\{D_A^{+1/2\,a}(\xi), T_B(\eta)\}_{\rm PB} = 8(-1)^A\delta_{AB}[D_A^{+1/2\,a}(\xi)\delta'(\xi-\eta) + \tfrac{1}{2}(D_A^{+1/2\,a})'\delta(\xi-\eta)]\,, \tag{14}$$

$$\{G_A^{+1/2\,a}(\xi), T_B(\eta)\}_{\rm PB} = 4(-1)^A\delta_{AB}[G_A^{+1/2\,a}(\xi)\delta'(\xi-\eta) + (G_A^{+1/2\,a})'\delta(\xi-\eta)]\,, \tag{15}$$

$$\{T_A(\xi), T_B(\eta)\}_{\rm PB} = 8(-1)^A\delta_{AB}[T_A(\xi)\delta'(\xi-\eta) + \tfrac{1}{2}(T_A)'\delta(\xi-\eta)]\,, \tag{16}$$

$$\{G_A^{+1/2\,a}(\xi), G_B^{+1/2\,b}(\eta)\}_{\rm PB} = {\rm i}\delta_{AB}C^{ab}\Pi_A^+\,\delta(\xi-\eta)\,, \tag{17}$$

with the following notations:

$$\psi_A^{+1/2\,a} \equiv (v^{+1/2}\sigma^a\theta'_A)\,, \quad r^{abc} \equiv v^{+1/2}\sigma^a\sigma^b\sigma^c v^{-1/2}\,, \quad R_A^{+1/2\,abc} = r^{adc}(v^{-1/2}\sigma^b\sigma_d\sigma^+\theta'_A)\,. \tag{18}$$

Let us now embed the system (1)–(3) into a larger system with additional gauge invariance such that the physical content of the original GS superstring is preserved. To this end we propose the following enlarged GS action (written once again in hamiltonian form):

$$\hat{S} = \int {\rm d}\tau \int_{-\pi}^{\pi} {\rm d}\xi\left(\mathscr{P}_\mu\partial_\tau X^\mu + \sum_A p^\alpha_{\theta A}\partial_\tau\theta_{A\alpha} + {\rm i}\sum_A \Xi^a_A\partial_\tau\Xi_{Aa} - \sum_A A_A\hat{T}_A - \sum_A A_A^{-1/2\,a}\hat{D}^{+1/2}_{Aa} - \sum_A M_A^{-1/2\,a}\hat{G}^{+1/2}_{Aa}\right)$$

$$+\hat{S}_{\rm harmonic}\,. \tag{19}$$

The new fermionic string variables $\Xi^a_A(\xi)$ obey according to (19) the canonical PB relations

$$\{\Xi^a_A(\xi), \Xi^b_B(\eta)\}_{\rm PB} = -{\rm i}C^{ab}\delta_{AB}\delta(\xi-\eta)\,.$$

All constraints in (19) are now first class only and read

$$\hat{T}_A = T_A + 2{\rm i}(-1)^A\Xi^a_A\Xi'_{Aa}\,, \tag{20}$$

$$\hat{D}_A^{+1/2\,a} = D_A^{+1/2\,a} - 2{\rm i}(-1)^A R_A^{+1/2\,abc}\Xi_{Ab}\Xi_{Ac}\,, \tag{21}$$

$$\hat{G}_A^{+1/2\,a} = G_A^{+1/2\,a} + (\Pi_A^+)^{1/2}\Xi^a_A\,, \tag{22}$$

where $R_A^{+1/2\,abc}$ is the same as in (18). The proof of the first-class property of the constraints (20)–(22) heavily relies on the Fierz identity (6). $\hat{S}_{\rm harmonic}$ in (19) is the same as $S_{\rm harmonic}$ (3) except for the constraint d^{ab} which is now modified to

$$\hat{d}^{ab} = d^{ab} + {\rm i}\sum_A \int_{-\pi}^{\pi} {\rm d}\xi\,\Xi^a_A(\xi)\Xi^b_A(\xi)\,. \tag{23}$$

The PB relations among the modified constraints (20)–(22) remain the same as the relations (13)–(16) for the old constraints (7), (9), (11) except for (12), (17):

$$\begin{aligned}\{\hat{D}_A^{+1/2\,a}(\xi), \hat{D}_B^{+1/2\,b}(\eta)\} &= -2{\rm i}C^{ab}\Pi_A^+\hat{T}_A\delta(\xi-\eta)\\ &-8(-1)^A\delta_{AB}\delta(\xi-\eta)[\psi_A^{+1/2\,a}\hat{D}_A^{+1/2\,b} + \psi_A^{+1/2\,b}\hat{D}_A^{+1/2\,a} - C^{ab}\psi_A^{+1/2\,c}\hat{D}^{+1/2}_{Ac}]\\ &+16(-1)^A\delta_{AB}\delta'(\xi-\eta)[r^{bcd}r^a{}_c{}^e - \tfrac{1}{2}C^{ab}C^{de}][(\Pi_A^+)^{1/2}\Xi_{Ad}\hat{G}^{+1/2}_{Ae}]\\ &+8(-1)^A\delta_{AB}\delta(\xi-\eta)r^{bcd}r^a{}_c{}^e({\rm d}/{\rm d}\xi)[(\Pi_A^+)^{1/2}\Xi_{Ad}]\hat{G}^{+1/2}_{Ae}\\ &-8(-1)^A\delta_{AB}\delta(\xi-\eta)r^{bce}r^a{}_c{}^d(\Pi_A^+)^{1/2}\Xi_{Ad}({\rm d}/{\rm d}\xi)\hat{G}^{+1/2}_{Ae} + 8(-1)^A\delta_{AB}\delta'(\xi-\eta)[r^{acd}r^b{}_c{}^e\hat{G}^{+1/2}_{Ad}(\xi)\hat{G}^{+1/2}_{Ae}(\xi)]\\ &+8(-1)^A\delta_{AB}\delta(\xi-\eta)[r^a{}_c{}^e r^{bcd}\hat{G}^{+1/2}_{Ae}({\rm d}/{\rm d}\xi)\hat{G}^{+1/2}_{Ad}]\,,\end{aligned} \tag{24}$$

$$\{\hat{G}_A^{+1/2\,a}(\xi), \hat{G}_B^{+1/2\,b}(\eta)\}_{\rm PB} = 0\,. \tag{25}$$

Similarly, the PB relations involving the modified harmonic constraints (23) do not change.

Clearly, the enlarged harmonic GS action (19) describes the same physical degrees of freedom as the original one (1). Indeed, the constrained system defined by (19) can be immediately reduced to the constrained system (1)–(3) after imposing the convariant gauge-fixing conditions $\Xi^a_A(\xi)=0$ corresponding to the new gauge invariances generated by the first-class constraints (22).

To conclude this section let us point out that we can covariantly disentangle the fermionic variables $\Xi^a_A(\xi)$ into mutually canonically conjugated pairs:

$$\varphi^k_A(\xi)=w^k_A\Xi^a_A(\xi),\quad \bar\varphi^k_A(\xi)=\bar w^k_a\Xi^a_A(\xi)\ ,$$

$$\{\varphi^k_A(\xi),\varphi^l_B(\eta)\}_{\rm PB}=\{\bar\varphi^k_A(\xi),\bar\varphi^l_B(\eta)\}_{\rm PB}=0,\quad \{\varphi^k_A(\xi),\bar\varphi^l_B(\eta)\}_{\rm PB}=-\mathrm{i}\delta_{AB}C^{kl}\delta(\xi-\eta)$$

by employing the second generation of harmonics w^k_a, $\bar w^k_a$ of refs. [2,3] which realize the coset space $SO(8)/SU(4)\times U(1)$:

$$w^k_a w^{la}=\bar w^k_a w^{la}=0,\quad w^k_a\bar w^{la}=C^{kl}\ .$$

Here C^{kl} denotes the $D=6$ charge-conjugation matrix and under the group $SU(4)\times U(1)$ of local rotations w^k_a, $\bar w^k_a$ transform as $(4,+1/2)$, $(\bar 4,-1/2)$, respectively.

3. BRST charge

According to the general BFV formalism [4,5] the BRST charge $Q_{\rm BRST}$ of constrained systems, whose PB algebra of first-class constraints possesses structure "constants" non-trivially depending on the canonical variables, may contain higher-order terms of power $2R+1$ ($R=2, 3, ...$) in the ghosts and the ghost momenta unlike the usual case $R=1$. The maximal value of R is called rank of the constrained system. The coefficient function in front of the ghost term of power $2R+1$ in $Q_{\rm BRST}$ is called Rth order structure function [4,5].

The above situation precisely occurs for the enlarged harmonic GS superstring [see the PB relations (24)]. Careful study of the general equations of refs.[4,5] (cf. e.g. eqs. (4.1.5), (4.1.6), (4.2.7), (4.2.8) of ref. [5]), recursively determining order by order in R the higher-order structure functions, show that in the present case all structure functions of order $R\geqslant 3$ identically vanish. However, we find the following non-zero second-order structure functions of the constrained system (19):

$$\overset{(2)}{U}{}_{DDD}{}^{GG},\quad \overset{(2)}{U}{}_{DDT}{}^{GG}\ . \tag{26}$$

In (26) we employ the condensed notations of refs.[4,5] where the indices T, D, G correspond to the constraints (20)–(22), respectively. Thus $Q_{\rm BRST}$ of (19) turns out to be of rank $R=2$.

The variables appearing in the expression for $Q_{\rm BRST}$ [eqs. (27)–(30) below] are organized as follows:

(i) constraints – $\hat T_A(\xi)$, $\hat D^{+1/2\,a}_A(\xi)$, $\hat G^{+1/2\,a}_A(\xi)$, $\hat D^{ab}$, D^{-+}, D_{+a};

(ii) ghosts – $c_A(\xi)$, $\chi^{-1/2\,a}_A(\xi)$, $\omega^{-1/2\,a}_A(\xi)$, η_{ab}, η^{+-}, η^{-a}.

It is convenient to divide $Q_{\rm BRST}$ into three pieces [#3]:

$$Q_{\rm BRST}=Q^{(1)}_{\rm string}+Q^{(2)}_{\rm string}+Q_{\rm harmonic}\ . \tag{27}$$

The last term on the RHS of (27) contains the contributions coming from the harmonic constraints (9) and (23):

[#3] Here we skip the trivial abelian part of $Q_{\rm BRST}$ involving bilinears in the momenta of the Lagrange multipliers and of the antighosts.

$$
\begin{aligned}
Q_{\text{harmonic}} = {} & \mathrm{i}\eta_{ab}\Big(\hat{D}^{ab} + \eta^{a}{}_{d}\,\partial/\partial\eta_{bd} - \eta^{b}{}_{d}\,\partial/\partial\eta_{ad} + \eta^{-a}\,\partial/\partial\eta^{-}{}_{b} - \eta^{-b}\,\partial/\partial\eta^{-}{}_{a} \\
& + \sum_{A}\int_{-\pi}^{\pi}\mathrm{d}\xi\,(\chi_A^{-1/2\,a}\,\delta/\delta\chi_{Ab}^{-1/2} - \chi_A^{-1/2\,b}\,\delta/\delta\chi_{Aa}^{-1/2} + \omega_A^{-1/2\,a}\,\delta/\delta\omega_{Ab}^{-1/2} - \omega_A^{-1/2\,b}\,\delta/\delta\omega_{Aa}^{-1/2})\Big) \\
& + \mathrm{i}\eta^{+-}\Big(D^{-+} - \eta^{-a}\,\partial/\partial\eta^{-a} - \tfrac{1}{2}\sum_{A}\int_{-\pi}^{\pi}\mathrm{d}\xi\,(\chi_A^{-1/2\,a}\,\delta/\delta\chi_A^{-1/2\,a} + \omega_A^{-1/2\,a}\,\delta/\delta\omega_A^{-1/2\,a})\Big) + \mathrm{i}\eta^{-}{}_{a}D^{+a}\,.
\end{aligned} \tag{28}
$$

The piece $Q^{(1)}_{\text{string}}$ in (27) contains the contributions of the string constraints (20)–(22) including the first-order structure functions:

$$
\begin{aligned}
Q^{(1)}_{\text{string}} = {} & \sum_{A}\int_{-\pi}^{\pi}\mathrm{d}\xi\,\{c_A\hat{T}_A + \chi_{Aa}^{-1/2}\hat{D}_A^{+1/2\,a} + [\omega_A^{-1/2\,a} - 2\mathrm{i}(-1)^A(\mathrm{d}/\mathrm{d}\xi)(\chi_A^{-1/2\,b}\chi_{Ab}^{-1/2}\,\delta/\delta\omega_{Aa}^{-1/2}) \\
& - 4\mathrm{i}(-1)^A r^{b}{}_{c}{}^{e} r^{dca}(\chi_{Ab}^{-1/2})'\chi_{Ad}^{-1/2}\,\delta/\delta\omega_A^{+1/2\,e}]\hat{G}_{Aa}^{+1/2}\} \\
& + 4\mathrm{i}\sum_{A}(-1)^A\int_{-\pi}^{\pi}\mathrm{d}\xi\,([c_A'c_A + \tfrac{1}{4}\mathrm{i}(-1)^A\Pi_A^{+}\chi_A^{-1/2\,a}\chi_{Aa}^{-1/2}]\,\delta/\delta c_A - \{c_A(\omega_A^{-1/2\,a})' + R_A^{+1/2\,cba}\chi_{Ac}^{-1/2}\omega_{Ab}^{-1/2} \\
& + 2r^{e}{}_{c}{}^{a}r^{bcd}(\Pi_A^{+})^{1/2}\Xi_{Ad}\chi_{Ae}^{-1/2}(\chi_{Ab}^{-1/2})' + (\mathrm{d}/\mathrm{d}\xi)[(\Pi_A^{+})^{1/2}\Xi_A^{a}]\chi_A^{-1/2\,b}\chi_{Ab}^{-1/2}\}\,\delta/\delta\omega_A^{-1/2\,a} \\
& + [c_A'\chi_A^{-1/2\,a} - c_A(\chi_A^{-1/2\,a})' + 2\psi_A^{+1/2\,b}\chi_{Ab}^{-1/2}\chi_A^{-1/2\,a} - \chi_A^{-1/2\,b}\chi_{Ab}^{-1/2}\psi_A^{+1/2\,a}]\,\delta/\delta\chi_A^{-1/2\,a})\,.
\end{aligned} \tag{29}
$$

Here once again the notations (18) were used. Note the unusual form of the term involving $\hat{G}_{Aa}^{+1/2}$ in (29) which is non-linear in the ghosts [it is due to the term quadratic in the constraints on the RHS of (24)].

Finally, the piece $Q^{(2)}_{\text{string}}$ in (27) incorporates the contribution of the non-vanishing second-order structure functions (26):

$$
\begin{aligned}
Q^{(2)}_{\text{string}} = {} & -4\sum_{A}\int_{-\pi}^{\pi}\mathrm{d}\xi\,\{\chi_A^{-1/2\,a}[\chi_{Aa}^{-1/2}(\chi_{Ab}^{-1/2})' - (\chi_{Aa}^{-1/2})'\chi_{Ab}^{-1/2}]\psi_A^{+1/2\,b}(\delta/\delta\omega_{Ac}^{-1/2})\,\delta/\delta\omega_A^{-1/2\,c} \\
& + 2\chi_A^{-1/2\,a}\chi_{Aa}^{-1/2}\chi_A^{-1/2\,c}R_{A\,c}^{+1/2\,de}(\delta/\delta\omega_A^{-1/2\,d})(\delta/\delta\omega_A^{-1/2\,e})'\} \\
& + \tfrac{4}{3}\sum_{A}\int_{-\pi}^{\pi}\mathrm{d}\chi\,\{c_A[\chi_A^{-1/2\,a}\chi_{Aa}^{-1/2}(\delta/\delta\omega_{Ab}^{-/2})'(\delta/\delta\omega_A^{-1/2\,b})' + \chi_A^{-1/2\,a}(\chi_{Aa}^{-1/2})'(\delta/\delta\omega_{Ab}^{-1/2})(\delta/\delta\omega_A^{-1/2\,b})'] \\
& - \tfrac{5}{2}c_A''\chi_A^{-1/2\,a}\chi_{Aa}^{-1/2}(\delta/\delta\omega_{Ab}^{-1/2})\,\delta/\delta\omega_A^{-1/2\,b}\}\,.
\end{aligned} \tag{30}
$$

Now, the explicit expression for Q_{BRST}, (27)–(30), provides the starting point for a manifestly superPoincaré-covariant second quantization of the GS superstring either in the BRST field theoretic formalism (17) or in the first-quantized function integral representation (the Polyakov formalism) [18]. The main conclusion from the present analysis is that the price to be paid for a manifestly superPoincaré-covariant quantization of the GS superstring is the appearance of fourth-order ghost terms in the BFV hamiltonian [4,5]:

$$
\begin{aligned}
H_{\text{BFV}} = \{Q_{\text{BRST}}, \Psi\} = {} & \sum_{A}\int_{-\pi}^{\pi}\mathrm{d}\xi\,\hat{T}_A - 4\mathrm{i}\sum_{A}(-1)^A\int_{-\pi}^{\pi}\mathrm{d}\xi\,[c_A'\,\delta/\delta c_A + (\chi_{Aa}^{-1/2})'\,\delta/\delta\chi_{Aa}^{-1/2} + (\omega_{Aa}^{-1/2})'\,\delta/\delta\omega_{Aa}^{-1/2}] \\
& + \tfrac{4}{3}\sum_{A}\int_{-\pi}^{\pi}\mathrm{d}\xi\,[\chi_A^{-1/2\,a}\chi_{Aa}^{-1/2}(\delta/\delta\omega_{Ab}^{-1/2})'(\delta/\delta\omega_A^{-1/2\,b})' + \chi_A^{-1/2\,a}(\chi_{Aa}^{-1/2})'(\delta/\delta\omega_{Ab}^{-1/2})(\delta/\delta\omega_A^{-1/2\,b})']\,.
\end{aligned} \tag{31}
$$

In (31) the BFV gauge-fixing function Ψ was chosen in the form $\Psi = \sum_a \partial/\partial c_{0A}$ with c_{0A} being the zero modes of the reparametrization ghosts $c_A(\xi)$.

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Note added

After submission of this paper it was realized [19] that, by taking as fermionic first-class constraints the following functionally independent combination:

$$\hat{\mathscr{D}}^{\alpha}_{A} = (\Pi^{+}_{A})^{-1}(\sigma^a v^{+1/2})^{\alpha} \hat{D}^{+1/2}_{Aa} + (\Pi^{+}_{A})^{-1}(\not{\Pi}_A \sigma^{+} \sigma^a v^{-1/2})^{\alpha} \hat{G}^{+1/2}_{Aa}$$

instead of the original first-class fermionic constraints $\hat{D}^{+1/2\,a}_{A}$ (21) and $\hat{G}_A{}^{+1/2\,a}$ (22), the PB algebra (24) and (25) radically simplifies and Q_{BRST} (27) becomes *first rank* (i.e. the higher ghost terms are absent). In the latter case also the manifest Parisi–Sourlas OSp(1, 1/2) symmetry [20] of H_{BFV} (31) was explicitly demonstrated in ref. [19].

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